Burlington County Institute of Technology

Medford Campus

Westampton Campus

Probability and Statistics

Department: Mathematics
Credits: 5
Revised: August 2023
Board Approval Date: August, 2023

Course Description

Statistics and statistical statements are widespread throughout our society, but few people possess more than a rudimentary understanding of their content or their implications. This lack of understanding can lead to unfavorable results, both financially and socially. For example, these common questions involve statistical concepts that an individual could experience on any given day:

What is a stronger statement, that some event "probably won't occur" or that it is "unlikely to occur"?
Why do so many people spend so much money purchasing lottery tickets?
How can you get caught in a downpour on a day with a forecast of a "0% chance of rain"?
What should you really believe when you see results of surveys and polls?
Are births of male babies and births of female babies equally likely to occur?
Which is safer, traveling to Florida on a bus or on a plane?

Probability and Statistics is a course designed to introduce students to basic and intermediate concepts of statistics, with an emphasis on understanding and interpreting real-world applications. Students will learn how to calculate probabilities, analyze events, make intelligent predictions, and test hypotheses and other claims from advertisements and political campaigns. Use of a calculator is required, along with basic algebraic manipulation of formulas, and geometric interpretations of areas. This course teaches concepts of which every adult citizen and consumer in our society should be familiar.

Table of Contents

Pacing Guide

Unit	Standards	Days
Unit 1: Introduction to Statistics	S-IC: 1, 3, 6	8
Unit 2: Descriptive Statistics	S-ID: 1, 2, 3, 5 S-IC: 1, 5	14
Unit 3: Probability	S-CP: 1, 2, 3, 4, 5, 6, 7, 8, 9	10
Unit 4: Discrete Probability	S-MD: 1, 2, 3, 4, 5, 6, 7	11
Unit 5: Normal Probability	S-MD: 1, 2, 3, 4, 5, 6, 7 S-ID: 4	10
Unit 6: Confidence Intervals	S-IC: 1, 2, 3, 4, 5, 6	15
Unit 7: Hypothesis Testing in One Sample	S-IC: 1, 2, 3, 4, 5, 6	10

Curriculum Maps

Unit 1: Introduction to Statistics

Desired Outcomes

Established Goals: NJSLS

S-IC Interpreting Categorical and Quantitative Data

- Understand and evaluate random processes underlying statistical experiments (S.IC.1).
- Make inferences and justify conclusions from sample surveys, experiments and observational studies (S.IC.3, S.IC.6).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ Statistics is the study of interpreting large amounts of data ○ Statistics attempts to determine to what extent data that applies to a sample can apply to a larger population. ○ Numerical data can be classified into different	○ What is statistics? ○ How do we distinguish between a population and a sample, a parameter and a statistic, descriptive and inferential statistics, and different data? ○ How do we classify with respect to the four levels of measurement?

types, with specified properties and characteristics ○ There are necessary requirements and standard practices to worthwhile statistical studies ○ If a study is not designed following those requirements and practices, the resulting data might be erroneous, misleading, or utterly worthless.	○ How do we design a statistical study, collect data, create a sample and identify a biased sample?
Students will know: ○ An Overview of Statistics ○ Data Classification ○ Experimental Design	**Students will be able to:** ○ Identify a population and sample. ○ Determine whether a numerical value describes a parameter or statistic. ○ Determine if data are qualitative or quantitative. ○ Identify a data set's level of measurement. ○ Decide which method of data collection you would use to collect data for a study. ○ Identify which sampling technique was used in a study. ○ Identify a bias or error that might occur in a survey or study.
Assessment Evidence	
Suggested Performance Tasks: ○ Real-world word problems with real data to be completed in a small group and class discussion to follow ⇒ Real Statistics Scenario: Research for technology publication ⇒ Case Study: Television Ratings	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments **Suggested Formative/Summative Assessments:** ○ Small group and full class presentations

⇒ Applet Activity: Random Numbers ⇒ Use and Abuses: Statistics in the Real World ○ Sectional DOLs	○ Student inquiries and projects ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Homework ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing

Learning Plan

Learning Activities:

Use a variety of instructional strategies such as:

- Inquiry-based Learning
- Problem-based Learning
- Project-based Learning
- Blended Learning
- Whiteboard Work
- Cooperative Learning
- Technology-based activities
- Do-Now Problems/Exit Problems
- Peer Editing

- Explicit Timing
- Guided Lecture

Related Standards

Interdisciplinary connections

- English Language Arts (RI.11-12.7) Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
  - ⇒ Example: Students will analyze the "Uses and Abuses: Statistics in the Real World" article on p. 28 of the textbook. Students will consider questions that arise in vaccine experiments with favorable and unfavorable results. Students will be asked to find a real-life experiment that had unfavorable results. What could have been done to avoid the outcome of the experiment? What are some problems that may arise when clinical trials of a new experimental drug or vaccine are stopped early and then the drug or vaccine is distributed to other subjects or patients?

Technology (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Example: Students will use Google Sheets to generate a list of random numbers, select sample members, and perform simulations.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
  - ⇒ Example: Students will develop lists of random numbers generated by the calculator. Students will explore their sets of numbers and engage in discussions of whether the random number generator is truly generating numbers randomly. Students will look at statistical models of their numbers and discuss whether "patterns" they see in the set of random numbers are possible if the numbers are truly random.

NJ SEL Competencies

- Self-Awareness: Recognize the impact of one's feelings and thoughts on one's own behavior
- Self-Management: Understand and practice strategies for managing one's own emotions, thoughts and behaviors

Climate Change

- Big Data and Climate Change: Students will explore the intersection of big data and climate change..

Culturally Relevant Connections

- Integrate Relevant Word Problems: Contextualize equations using word problems that reference student interests and cultures. Example: When learning about which type of data collection to employ, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:	**ELL:**

- Encourage students to justify their reasoning. Provide students with sentence stems if needed.
- Conceptual word wall that contains definition, translation, pictures and/or examples
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Teacher modeling.
- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily
- Develop a reference document with students with verbal and pictorial descriptions.

- Sentence stems to provide additional language support.
- The students can explain through a “think aloud” and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use of teacher created reference sheets during task completion to check expectations, verify content and support language acquisition.
- Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary.
- Identify key phrases or new vocabulary to pre-teach.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference

Enrichment

- Extended learning goals:
  - ⇒ Extend WS from Resource sets
  - ⇒ Article Analysis: Students write up a summary of a magazine or newspaper article that involves a statistical study. They are asked to identify the key components of the study (who, what, where, why, how, etc.) and look for examples of misrepresentations or poor reasoning. More stock is put into their writing as they have just scratched the surface of the course material.

Unit 2: Descriptive Statistics

Desired Outcomes

Established Goals: NJSLS

S-ID Interpreting Categorical and Qualitative Data
- Summarize, represent, and interpret data on a single count or measurement variable (S.ID.1, S.ID.2, S.ID.3).
- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.5).

S-IC Making Inferences and Justifying Conclusions
- Understand and evaluate random processes underlying statistical experiments (S.IC.1).
- Make inferences and justify conclusions from sample surveys, experiments and observational studies (S.IC.5).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ Frequency distributions are collections and depictions of the different results of real-world phenomena. ○ Descriptive Statistics is focused on the varied processes of describing qualities of large amounts of data with one or more numbers	○ How do we construct a frequency distribution? ○ How do we construct different types of graphs to present and describe data? ○ How do we find the mean, median, mode, and range of a population and a sample? ○ How do we find the variance and standard

	deviation of a population and a sample? ○ How do we use the Empirical Rule to interpret standard deviation? ○ How do we find the first, second, third quartiles, and interquartile range of a data set? ○ How do we find and interpret the standard score(z-score)?
Students will know: ○ Frequency Distributions and Their Graphs ○ More Graphs and Displays ○ Measures of Central Tendency ○ Measures of Variation ○ Measures of Position	**Students will be able to:** 1a. Use a given frequency distribution to find the class width, midpoints, and boundaries. b. Use a frequency histogram to determine the number of classes, estimate the frequency of a class with the least and greatest frequency, and determine the class width. c. Use an o give to approximate the number in a sample and the location of the greatest increase in frequency. d. Use relative frequency histogram/polygon to identify the class with the greatest and least relative frequency, approximate the greatest and least relative frequency, and frequency of the second class. e. Construct a frequency, cumulative/relative frequency distribution/histogram/polygon and o given for the data set using the indicated number of classes. 2a. Make a dot plot/stem-and-leaf plot for a data set, and identify the minimum and maximum entries. b. Make conclusions from various graphs. c. Organize data using various graphs. 3a. Determine whether the approximate shape of distribution in a histogram is symmetric, uniform,

	skewed left, skewed right, or none of these. b. Find the mean, median, mode of a data set. If none can be found, explain why. c. Determine which measure best represents the data. d. Find the weighted mean of a data set. e. Approximate the mean of grouped data. 4a. Find the range, mean, variance, standard deviation, and range of a population/sample data set. b. Compare two and three data sets. c. Use the empirical rule. d. Find a grouped mean and standard deviation for a data set. 5a. Find the three quartiles/draw a box-and-whisker plot for a data set. b. Use a box-and-whisker plot to identify different values. c. Use a calculator to find the three quartiles/draw a box-and-whisker plot for a data set. d. Label z-scores as usual, unusual, or very unusual. e. Transform an x-value to a z-score and compare to other z-scores. f. Interpret different percentiles.
Assessment Evidence	
<u>Suggested Performance Tasks:</u> ○ Real-world word problems with real data to be completed in a small group and class discussion to follow ⇒ Applet Activity: Mean vs. Median; Standard Deviation	**<u>Required District/State Assessments:</u>** ○ Unit Assessment ○ SGO Assessments **<u>Suggested Formative/Summative Assessments:</u>**

⇒ Case Study: Business Size ⇒ Uses and Abuses: Statistics in the Real World ⇒ Real Statistics: Monthly Rental Fees ⇒ Technology Investigation: Parking Tickets ○ Sectional DOLs	○ Small group and full class presentations ○ Student inquiries and projects ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Homework ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing

Learning Plan

Learning Activities:

Use a variety of instructional strategies such as:

- Inquiry-based Learning
- Problem-based Learning
- Project-based Learning
- Blended Learning
- Whiteboard Work
- Cooperative Learning
- Technology-based activities
- Do-Now Problems/Exit Problems

- Peer Editing
- Explicit Timing
- Guided Lecture

Related Standards

Interdisciplinary connections

- Science (HS-ESS3-1, HS-ESS3-5)
  - ⇒ Billion-Dollar Weather and Climate Disasters: Students will analyze the National Oceanic and Atmospheric Administration details of the financial costs and frequencies of extreme weather disasters exceeding one billion dollars in the US since 1980.

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena
  - ⇒ Example: Analyze sets of data in Google Sheets. Use spreadsheet formulas and chart tools to find the sample mean, sample standard deviation, five-number summary, frequency distribution, and create a histogram.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Example: Students will construct graphs for frequency distributions on Excel/Google Sheets, and TI-84.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
  - ⇒ Central Tendency of College Costs: College expenses range from tuition to housing to school supplies. In this activity, you will take a look at how much these college expenses can add up and calculate the measures of central tendency for college costs.

NJ SEL Competencies

- Social Awareness: Recognize and identify the thoughts, feelings and perspectives of others
- Responsible Decision-Making: Develop, implement and model effective problem solving and critical thinking skills

Climate Change

- Billion-Dollar Weather and Climate Disasters: Students will analyze the National Oceanic and Atmospheric Administration details of the financial costs and frequencies of extreme weather disasters exceeding one billion dollars in the US since 1980.

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare frequency distributions and their graphs by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:	**ELL:**
○ Encourage students to justify their reasoning. Provide students with sentence stems if needed. ○ Anchor charts to model strategies and process ○ Reference sheets that list formulas, step-by-step procedures and model strategies ○ Conceptual word wall that contains definition, translation, pictures and/or examples ○ Highlight and label solution steps for multi-step problems in different colors ○ Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily ○ Videos to reinforce skills and thinking behind concepts ○ Access to tools such as tables, graphs and charts to solve problems ○ Provide students with a graphic organizer that outlines the possible solution paths, formulas and sample problems to facilitate independence. ○ Encourage students to verbalize their thinking while solving problems by asking, assessing and advancing questions.	○ Assign a buddy, same language or English speaking ○ Allow errors in speaking ○ Accept participation at any level, even one word ○ Sentence stems to provide additional language support ○ Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills. ○ The students can explain through a “think aloud” and demonstrate how they solved the problem. ○ Use of teacher created reference sheets during task completion to check expectations, verify content and support language acquisition. ○ Provide students with visuals aids like pictures and diagrams to illustrate the parts of an expression. ○ Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary. ○ Identify key phrases or new vocabulary to pre-teach. ○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.

Enrichment

- Extended learning goals:
  - ⇒ Extend WS from Resource sets
  - ⇒ Nielsen Data Project: Students input the Nielsen ratings from the current week of television shows. They use Google Sheets to create appropriate charts, graphs, and summary statistics. They then write a paper comparing the viewership of the different television networks. Students then create a five minute presentation to an advertising executive to tell what stations, shows, times, or days of week bring in the most viewers.
  - ⇒ Health/Sports/Science: Collect data on pulse rates over several days and create a frequency distribution, a frequency histogram, and calculate mean, median, mode, variance, standard deviation and quartiles.

Unit 3: Probability

Desired Outcomes

Established Goals: NJSLS

S-CP Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data (S.CP.1, S.CP.2, S.CP.3, S.CP.4, S.CP.5).
- Use the rules of probability to compute probabilities of compound events in a uniform probability model (S.CP.6, S.CP.7, S.CP.8, S.CP.9).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:

Students will understand that...

- All probabilities are based upon distributions of outcomes, and all probability questions can be answered by a careful analysis of the listing of all possible outcomes

Essential Questions:

- How do we identify the sample space and distinguish among the 3 types of probability?
- How do we find conditional probabilities and distinguish between independent and dependent events?
- How do we find the probability of two events occurring in sequence or just simply occurring

	and determine mutual exclusiveness? ○ How do we find the number of ways two or more events can occur, a group of objects can be arranged in order, and to choose several objects from a group without regard to order?
Students will know: ○ Basic Concepts of Probability ○ Conditional Probability and the Multiplication Rule ○ The Addition Rule ○ Counting Principles	**Students will be able to:** 1a. Identify a sample space and list the outcomes of an event 1b. Classify a statement as an example of classical, empirical, or subjective probability 1c. Use a table to determine the probability of an event 2a. Find a conditional probability from a given statement 2b. Decide whether events are dependent or independent 2c. Find the probability of a sequence of events 3a. Decide if events are mutually exclusive 3b. Find normal probabilities 3c. Find probabilities from pie/pareto charts 4a. Use the fundamental counting principle 4b. Use combinations and permutations 4c. Use other counting principles to find probabilities
Assessment Evidence	
Suggested Performance Tasks: ○ Real-world word problems with real data to be completed in a small group and class discussion	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments

to follow ⇒ Applet Activity: Simulating the Stock Market ⇒ Case Study: United States Congress ⇒ Uses and Abuses: Statistics in the Real World ⇒ Real Statistics: Password Security ⇒ Technology Simulations: Composing Mozart Simulations with Dice ○ Sectional DOLs	**Suggested Formative/Summative Assessments:** ○ Small group and full class presentations ○ Student inquiries and projects ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Homework ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing

Learning Plan

Learning Activities:

Use a variety of instructional strategies such as:

- Inquiry-based Learning
- Problem-based Learning
- Project-based Learning
- Blended Learning
- Whiteboard Work

- Cooperative Learning
- Technology-based activities
- Do-Now Problems/Exit Problems
- Peer Editing
- Explicit Timing
- Guided Lecture

Related Standards

Interdisciplinary connections

- Social Studies: 6.1.12.CivicsDP.6.a
  - ⇒ Examples: Students will analyze the demographics of Congress by calculating various probabilities from tables showing the makeup of Congress by gender and political party (use questions from Chapter 3 case study, but have students research and tabulate current tables).

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
  - ⇒ Example: Chapter 3 Technology Simulation - Composing Mozart Simulations with Dice

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.1.12.RM.3: Compare the cost of various types of insurance (e.g., life, homeowners, motor vehicle) for the same product or service, strategies to lower costs, and the process for filing an insurance claim.
  - ⇒ Example: Insuring multiple events with compound probability - Students will be able to explore how redundancy helps failure rates of a system, how insurance companies set premiums, and how insurance companies use the Law of Large Numbers.

NJ SEL Competencies

- Relationship Skills: Establish and maintain healthy relationships

- Self-Awareness: Recognize one's personal traits, strengths and limitations

Culturally Relevant Connections

- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and /or the community. Example: Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Teacher modeling.
- Highlight and label solution steps for multi-step problems in different colors
- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily
- Videos to reinforce skills and thinking behind concepts
- Provide students with a graphic organizer that

ELL:

- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a "think aloud" and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Create an outline that allows students to organize and follow information that they are receiving. Outlines can be blank or partially filled in to vary difficulty.
- Provide students with visuals aids like pictures and diagrams to illustrate the parts of an

outlines the possible solution paths, formulas and sample problems to facilitate independence. ○ Encourage students to verbalize their thinking while solving problems by asking, assessing and advancing questions. ○ Provide students with reference sheets/notes to encourage confidence and independence.	expression. ○ Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary. ○ Identify key phrases or new vocabulary to pre-teach. ○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.

Enrichment

- Extended learning goals:
  - ⇒ Simulating the Stock Market (Textbook p. 146) - This applet allows students to investigate the probability that the stock market will go up on any given day.
  - ⇒ Simulation: Composing Mozart Variations with Dice (Textbook p. 187)

Unit 4: Discrete Probability Distributions

Desired Outcomes

Established Goals: NJSLS

- Calculate expected values and use them to solve problems (S.MD.1, S.MD.2, S.MD.3, S.MD.4).
- Use probability to evaluate outcomes of decisions (S.MD.5, S.MD.6, S.MD.7).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:

- Probability Distributions are based upon the possible outcomes of an experiment or an event.
- Random variables are necessary to quantify and describe characteristics of probability distributions
- Different types of real-world phenomena are modeled by different types of probability distributions

Essential Questions:

- How do we distinguish between random variables?
- How do we determine if a distribution is a probability or binomial distribution, construct the appropriate distribution, and its graph?
- How do we find the mean, variance, and standard deviation of the distribution?
- How do we determine if an experiment is binomial, geometric, or Poisson and find its probabilities?

Students will know: 1. Probability Distributions 2. Binomial Distributions 3. More Discrete Probability Distributions	**Students will be able to:** 1a. Decide whether a random variable is discrete or continuous 1b. Decide whether a distribution is a probability distribution 1c. Construct a probability distribution; graph using a histogram; and find the mean (expected value), variance, and standard deviation 2a. Decide whether an experiment is a binomial experiment; list the values of n, p, q, and x 2b. Construct a binomial distribution; graph using a histogram; and find the mean (expected value), variance, and standard deviation 3a. Find probabilities using the geometric distribution (use technology) 3b. Find probabilities using the Poisson distribution
Assessment Evidence	
Suggested Performance Tasks: ○ Real-world word problems with real data to be completed in a small group and class discussion to follow ⇒ Applet Investigation: Binomial Distribution ⇒ Case Study: Distribution of Number of Hits in Baseball Games ⇒ Uses and Abuses: Statistics in the Real	**Required District/State Assessments:** ○ Unit Assessment ○ SGO Assessments **Suggested Formative/Summative Assessments:** ○ Small group and full class presentations ○ Student inquiries and projects

World ⇒ Real Statistics: ART Results ⇒ Technology Investigation: Using Poisson Distributions as Queuing Models ○ Sectional DOLs	○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Homework ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing

Learning Plan

Learning Activities:

- Inquiry-based Learning
- Problem-based Learning
- Project-based Learning
- Blended Learning
- Whiteboard Work
- Cooperative Learning
- Technology-based activities
- Do-Now Problems/Exit Problems
- Peer Editing
- Explicit Timing
- Guided Lecture

Related Standards

Interdisciplinary connections

- Science (HS-LS3-3) Apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population.
  - ⇒ Example: According to a theory in genetics, if tall and colorful plants are crossed with short and colorless plants, four types of plants will result: tall and colorful, tall and colorless, short and colorful, and short and colorless, with corresponding probabilities of 9/16, 3/16, 3/16 and 1/16. If 10 plants are selected, find the probability that 5 will be tall and colorful, 2 will be tall and colorless, 2 will be short and colorful, and 1 will be short and colorless.
  - ⇒ Example: Another proposed theory in genetics gives the corresponding probabilities for the four types of plants described in the latter example as 5/16, 4/16, 1/16, and 1/16. If 10 plants are selected, find the probability that 5 will be tall and colorful, 2 will be tall and colorless, 2 will be short and colorful, and 1 will be short and colorless.

Technology (NJSLS Computer Science and Design Thinking)

- 8.2.12.ED.5: Evaluate the effectiveness of a product or system based on factors that are related to its requirements, specifications, and constraints (e.g., safety, reliability, economic considerations, quality control, environmental concerns, manufacturability, maintenance and repair, ergonomics).
  - ⇒ Example: Students will read, analyze, and apply the Uses and Abuses: Statistics in the Real World article from Chapter 4 of the text (p.223) examining binomial probabilities in manufacturer product claims and ethics.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Example: Chapter 4 Technology Lab on Using Poisson Distributions as Queuing Models in Excel/Google Spreadsheet (p. 231)

NJ SEL Competencies

- Self-Management: Recognize the skills needed to establish and achieve personal and educational goals
- Social Awareness: Demonstrate an understanding of the need for mutual respect when viewpoints differ

Culturally Relevant Connections

- Integrate Relevant Word Problems: Contextualize equations using word problems that reference student interests and cultures. Example: When learning about probability distributions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare binomial distributions by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Encourage students to justify their reasoning.

ELL:

- The students can explain through a “think aloud” and demonstrate how they solved the problem.

Provide students with sentence stems if needed. ○ Anchor charts to model strategies and process ○ Conceptual word wall that contains definition, translation, pictures and/or examples ○ Teacher modeling. ○ Highlight and label solution steps for multi-step problems in different colors ○ Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily ○ Provide students with a graphic organizer that outlines the possible solution paths, formulas and sample problems to facilitate independence. ○ Encourage students to verbalize their thinking while solving problems by asking, assessing and advancing questions. ○ Provide students with reference sheets/notes to encourage confidence and independence.	○ Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit). ○ Use of teacher created reference sheets during task completion to check expectations, verify content and support language acquisition. ○ Provide students with visuals aids like pictures and diagrams to illustrate the parts of an expression. ○ Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary. ○ Identify key phrases or new vocabulary to pre-teach. ○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems.

Enrichment

- Extended learning goals:
  - ⇒ Extend WS from Resource sets
  - ⇒ Business: Use a binomial probability distribution to determine the optimal number of reservations to book on a flight that has 168 seats using the experimental probability of a person with a reservation not showing up.

Unit 5: Normal Probability Distributions

Desired Outcomes

Established Goals: NJSLS

S-MD Using Probability to Make Decisions
- Calculate expected values and use them to solve problems (S.MD.1, S.MD.2, S.MD.3, S.MD.4).
- Use probability to evaluate outcomes of decisions (S.MD.5, S.MD.6, S.MD.7).

S-ID Categorical and Quantitative Data
- Summarize, represent, and interpret data on a single count or measurement variable (S.ID.4).

NJSLS Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ The probability and area under the normal distribution curve are synonyms, the data can be expressed as raw or standardized. ○ Data of different sample sizes must be treated differently, and the data set will have to be corrected for mean error and continuity	○ How do we interpret graphs of normal probability distributions? ○ How do we find/interpret z-scores and transform a z-score to an x-value? ○ How do we find areas under the standard normal curve, probabilities for normally distributed

correction.	variables, and specific data values of a normal distribution? ○ How do we find a sampling distribution? ○ How do we interpret the Central Limit Theorem and apply it? ○ How do we decide when the normal distribution can be used to approximate the binomial and then use it?
Students will know: 1. Introduction to Normal Distributions and the Standard Normal Distribution 2. Normal Distributions: Finding Probabilities 3. Normal Distributions: Finding Values 4. Sampling Distributions and the Central Limit Theorem 5. Normal Approximations to Binomial Distributions	**Students will be able to:** 1a. Use a graph to estimate the mean and standard deviation 1b. Use the standard normal table to find probabilities under the curve 2a. Find probabilities from known z-scores 2b. Find probabilities from known x-values 3a. Find z-scores from given probabilities and percentiles 4a. Use a given population to find a sampling distribution of sample means for indicated sample sizes; find the mean and standard deviation of the population and also of the sampling distribution to compare 4b. Use the central limit theorem to find the mean and standard deviation of the mean; sketch a graph of the sampling distribution 4c. Find probabilities from sampling distributions 5a. Decide whether you can use the normal distribution to approximate the binomial; find the mean and standard deviation

	5b. Write a binomial probability as a normal probability using the continuity correction 5c. Use the normal distribution to approximate probabilities and sketch their graphs; if you cannot, use the binomial

Assessment Evidence

Suggested Performance Tasks:

- Real-world word problems with real data to be completed in a small group and class discussion to follow
  - ⇒ Case Study: Birth Weights in America
  - ⇒ Applet Investigation: Sampling Distributions
  - ⇒ Uses and Abuses: Statistics in the Real World
  - ⇒ Real Statistics: Pharmaceutical Machine Calibration
- Sectional DOLs

Required District/State Assessments:

- Unit Assessment
- SGO Assessments

Suggested Formative/Summative Assessments:

- Small group and full class presentations
- Student inquiries and projects
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Homework
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations

	○ Role Playing

Learning Plan

Learning Activities:

- Inquiry-based Learning
- Problem-based Learning
- Project-based Learning
- Blended Learning
- Whiteboard Work
- Cooperative Learning
- Technology-based activities
- Do-Now Problems/Exit Problems
- Peer Editing
- Explicit Timing
- Guided Lecture

Related Standards

Interdisciplinary connections

- Science (HS-ESS2-4, HS-ESS3-5)
  - ⇒ Averages, Variability, and Extremes in Weather and Climate: Students will explore how global temperature distribution can be represented by a bell curve. When temperatures become more variable, the bell curve widens, and when average temperatures increase, the bell curve moves further down the axis.
- English Language Arts (RI.11-12.7)
  - ⇒ Example: You work for a pharmaceutical company as a statistical process analyst. Your job is to analyze processes and make sure they are in statistical control. In one process, a machine is supposed to add 9.8 milligrams of a compound to a mixture in a vial. (Assume this process can be approximated by a normal distribution.) The acceptable range of amounts of the compound added is 9.65 milligrams to 9.95 milligrams, inclusive. Because of an error with the release valve, the setting on the machine "shifts" from 9.8 milligrams. To check that the machine is adding the correct amount of the compound

into the vials, you select at random three samples of five vials and find the mean amount of the compound added for each sample. A coworker asks why you take 3 samples of size 5 and find the mean instead of randomly choosing and measuring the amounts in 15 vials individually to check the machine's settings. (Note: Both samples are chosen without replacement.) Write a paragraph to your coworker explaining why you take 3 samples of size 5 and find the mean of each sample instead of randomly choosing and measuring the amounts in 15 vials individually to check the machine's setting.

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena
  - ⇒ Example: Complete the Chapter 5 Technology Lab pertaining to the US Census Bureau's 2011 data on the age of adults in the US. Use Excel or a Google Spreadsheet to organize your results to the exercises on multiple named worksheets/tabs within the spreadsheet workbook.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
  - ⇒ Example: Students will be able to calculate probabilities using a variety of technology, including spreadsheets and a graphing calculator.

NJ SEL Competencies

- Responsible Decision-Making: Identify the consequences associated with one's actions in order to make constructive choices
- Relationship Skills: Identify ways to resist inappropriate social pressure

Climate Change

- Averages, Variability, and Extremes in Weather and Climate: Students will explore how global temperature distribution can be represented by a bell curve. When temperatures become more variable, the bell curve widens, and when average temperatures increase, the bell curve moves further down the axis.

Culturally Relevant Connections

- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn how to use the normal distribution to approximate probabilities by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
- Integrate Relevant Word Problems: Contextualize equations using word problems that reference student interests and cultures. Example: When learning about the significance of the different elements of the equation, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct and compare linear, quadratic and exponential models by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Anchor charts to model strategies and process
- Reference sheets that list formulas, step-by-step procedures and model strategies
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context

ELL:

- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a “think aloud” and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words

○ Highlight and label solution steps for multi-step problems in different colors ○ Videos to reinforce skills and thinking behind concepts ○ Access to tools such as tables, graphs and charts to solve problems ○ Provide students with a graphic organizer that outlines the possible solution paths, formulas and sample problems to facilitate independence. ○ Encourage students to verbalize their thinking while solving problems by asking, assessing and advancing questions.	for reference (from a specific unit). ○ Use of teacher created reference sheets during task completion to check expectations, verify content and support language acquisition. ○ Provide students with visuals aids like pictures and diagrams to illustrate the parts of an expression. ○ Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary. ○ Identify key phrases or new vocabulary to pre-teach. ○ Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.

Enrichment

- Extended learning goals:
  - ⇒ Extend WS from Resource sets
  - ⇒ Biology: Examine a data set for the lengths of petals of Wild Irises and verify the Central Limit Theorem by calculating probabilities.

Unit 6: Confidence Intervals

Desired Outcomes

Established Goals: NJSLS

S-IC Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments (S.IC.1, S.IC.2).
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S.IC.3, S.IC.4, S.IC.5, S.IC.6).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:

- With a specified level of confidence, intervals will be created and interpreted for the mean, proportion, variance, and standard deviation.
- The minimum sample size is important to all aspects of statistics and will be computed and discussed to connect back to what constitutes a good statistical study.

Essential Questions:

- How do we find a point estimate and margin of error?
- How do we construct and interpret confidence intervals for the population mean/proportion/standard deviation with different sample sizes and also when the population standard deviation is known and unknown?

	○ How do we determine the minimum sample size needed when estimating the population mean/proportion/standard deviation?
Students will know: 1. Confidence Intervals for the Mean (Large Samples) 2. Confidence Intervals for the Mean (Small Samples) 3. Confidence Intervals for Population Proportions 4. Confidence Intervals for Standard Deviation	**Students will be able to:** 1a. Find the point estimate (sample mean) of a population mean and the margin of error for a 90% confidence interval 1b. Construct a confidence interval for a population mean 1c. Determine the minimum sample size needed to estimate a population mean 2a. Find a t-critical value for a given confidence level and sample size 2b. Find the margin of error for the population mean 3a. Use given information or pie charts to find p-hat and q-hat 3b. Construct a confidence interval for a population proportion 4a. Find the chi-square critical values needed to estimate the population variance for a given confidence level and sample size 4b. Construct a confidence interval for the population
Assessment Evidence	
Suggested Performance Tasks:	**Required District/State Assessments:**

- Real-world word problems with real data to be completed in a small group and class discussion to follow
  - ⇒ Applet Investigation: Confidence Intervals for a Mean; Confidence Intervals for a Proportion
  - ⇒ Case Study: Marathon Training
  - ⇒ Uses and Abuses: Statistics in the Real World
  - ⇒ Real Statistics: Contaminants in Drinking Water
- Sectional DOLs

- Unit Assessment
- SGO Assessments

Suggested Formative/Summative Assessments:

- Small group and full class presentations
- Student inquiries and projects
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Homework
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing

Learning Plan

Learning Activities:

- Inquiry-based Learning
- Problem-based Learning
- Project-based Learning
- Blended Learning

- Whiteboard Work
- Cooperative Learning
- Technology-based activities
- Do-Now Problems/Exit Problems
- Peer Editing
- Explicit Timing
- Guided Lecture

Related Standards

Interdisciplinary connections

- English Language Arts RL.9-10.3
  - ⇒ Example: Students analyze book ratings from an online platform to explore the development of complex characters in literature. By calculating and interpreting confidence intervals for average book ratings, students determine the reliability and precision of readers' opinions on character development. This project addresses the New Jersey Student Learning Standard for English Language Arts (RL.9-10.3) by integrating statistical analysis with literary analysis, emphasizing the importance of understanding character development over the course of a text. Through data collection, calculations, analysis, and presentations, students gain insights into the relationship between statistics and literature, enhancing their critical thinking and analytical skills.

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
  - ⇒ Example: Students will complete Activity 6.3 (p. 329) in which they use an Applet to run simulations and calculate confidence intervals. They will organize their results and analysis on different tabs/worksheets within one workbook spreadsheet using Excel or Google Sheets.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.1.12.RM.1: Describe the importance of various sources of income in retirement, including Social Security,

employer-sponsored retirement savings plans, and personal investments.

⇒ Example: Find the year-to-date percent return for a sample of 50 mutual funds at http://finance.yahoo.com/funds/lists/ . Then determine a 95% and a 99% confidence interval for the mean annual percent return for all mutual funds and interpret.

NJ SEL Competencies

- Self-Awareness: Recognize the importance of self-confidence in handling daily tasks and challenges
- Self-Management: Identify and apply ways to persevere or overcome barriers through alternative methods to achieve one's goals

Climate Change

- Critical Values & Margins of Error: This lesson applies information about sample size, critical values, standard error, and confidence intervals to analyze a study about the safety of tap water in Flint, Michigan.

Culturally Relevant Connections

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn how to create confidence intervals by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Reference sheets that list formulas, step-by-step procedures and model strategies
- Highlight and label solution steps for multi-step problems in different colors
- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily
- Videos to reinforce skills and thinking behind concepts
- Access to tools such as tables, graphs and charts to solve problems
- Provide students with a graphic organizer that outlines the possible solution paths, formulas and sample problems to facilitate independence.
- Encourage students to verbalize their thinking while solving problems by asking, assessing and advancing questions.
- Provide students with reference sheets/notes to encourage confidence and independence.

ELL:

- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a "think aloud" and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use of teacher created reference sheets during task completion to check expectations, verify content and support language acquisition.
- Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.

Enrichment

- Extended learning goals:
  - ⇒ Extend WS from Resource sets
  - ⇒ Confidence Interval Project: Students will develop a question about the population of students at their high school. After receiving approval for their question, they will use a random number generator to randomly select a sample of students to survey regarding their question. Using their collected data, they determine the p-hat value, if conditions were met to form a 95% confidence interval, and express

their findings in a presentation.

Unit 7: Hypothesis Testing in One Sample

Desired Outcomes

Established Goals: NJSLS

S-IC Making Inferences and Justifying Conclusions
- Understand and evaluate random processes underlying statistical experiments (S.IC.1, S.IC.2).
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S.IC.3, S.IC.4, S.IC.5, S.IC.6).

NJSLS Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:

- The relationship between the null and alternative hypothesis is crucial in creating the normal distribution as well as the rejection region and the fail to reject region.
- The type of testing needed will be gleaned from the word problems using different strategies taught in class.

Essential Questions:

- How do we state the null and alternative hypotheses?
- How do we interpret the level of significance for a hypothesis test, identify type I and type II errors, know whether to use a one-tailed or two-tailed test, and interpret a decision based on the results of a test?
- How do we find critical values for a z, t, proportion,

	and Chi-Square test? ○ How do we use a z, t, proportion, or Chi-Square test to test a population parameter? ○ How do we find p-values/rejection regions by hand and with technology then use them in a test?
Students will know: 1. Introduction to Hypothesis Testing 2. Hypothesis Testing for the Mean (Large Samples) 3. Hypothesis Testing for the Mean (Small Samples) 4. Hypothesis Testing for Proportions 5. Hypothesis Testing for Standard Deviation	**Students will be able to:** 1a. Use a given claim to state a null and alternative hypothesis; identify which is the claim 1b. Identify type I and II errors for a hypothesis test 1c. Determine if a test is left, right, or two tailed 1d. Interpret a decision that rejects/fails to reject the null 2a. Find a critical value for a given z-test and and level of significance 2b. Use a z-test to test the claim about a population mean at a given level of significance with given sample statistics 2c. Use a P-value to test the claim about a population mean using given sample statistics; state your decision for different levels of significance 2d. Test the claim about the population mean using the rejection region method 3a. Find the critical value for the indicated t-test, level of significance and sample size 3b. Use a t-test to test the claim about the population mean at the given level of significance using given sample statistics, assume the population is normally distributed

	4a. Decide whether the normal distribution can be used to approximate the binomial, then use a z-test to test a claim about the population proportion at a given level of significance with given sample statistics 5a. Find the critical for a chi-squared test for a population standard deviation at a given level of significance with given sample statistics 5b. Use a chi-squared test to test the claim about a population standard deviation at a given level of significance with given sample statistics assuming normally distributed.

Assessment Evidence

Suggested Performance Tasks:

- Real-world word problems with real data to be completed in a small group and class discussion to follow
  - ⇒ Applet Investigation: Hypothesis Tests for a Mean; Hypothesis Tests for a Proportion
  - ⇒ Case Study: Human Body Temperature
  - ⇒ Uses and Abuses: Statistics in the Real World
  - ⇒ Real Statistics: Cola Taste Test
  - ⇒ Technology Investigation: The Case of the Vanishing Women
- Sectional DOLs

Required District/State Assessments:

- Unit Assessment
- SGO Assessments

Suggested Formative/Summative Assessments:

- Small group and full class presentations
- Student inquiries and projects
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor

	○ Quizzes ○ Homework ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing

Learning Plan

Learning Activities:

- Inquiry-based Learning
- Problem-based Learning
- Project-based Learning
- Blended Learning
- Whiteboard Work
- Cooperative Learning
- Technology-based activities
- Do-Now Problems/Exit Problems
- Peer Editing
- Explicit Timing
- Guided Lecture

Related Standards

Interdisciplinary connections

- Health and Physical Education (2.3.12.HCDM.4): Evaluate emerging methods to diagnose and treat diseases and health conditions that are common in young adults in the United States and in other countries (e.g., hepatitis, stroke, heart attacks, cancer,).
  - ⇒ Example: Students will complete a Case Study Analysis on human body temperature based on an

article in the Journal of Statistics Education (vol. 4, no. 2). Students will complete hypothesis tests and analyze possible sources of error in Wunderlich's sampling procedure established over 100 years ago. Students will analyze how the accepted mean body temperature can impact the diagnosis of health conditions that present with abnormal body temperatures.

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
  - ⇒ Example: Students will complete Activity 7.3 (p. 386) in which they use a hypothesis tests for a mean applet to visually investigate hypothesis tests for a mean. They will organize their findings and analysis on different tabs/worksheets within one workbook spreadsheet using Excel or Google Sheets.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
  - ⇒ Example: Students will compare using the TI-84 Plus calculator and Google Sheets to set up a hypothesis test and interpret the results within the context of the problem.

NJ SEL Competencies

- Social Awareness: Demonstrate an understanding of the need for mutual respect when viewpoints differ
- Responsible Decision-Making: Evaluate personal, ethical, safety and civic impact of decisions

Climate Change

- WATER Students apply information about sample size, critical values, standard error, and confidence intervals to analyze a study about the safety of tap water in Flint, Michigan.

Culturally Relevant Connections

- Integrate Relevant Word Problems: Contextualize equations using word problems that reference student

interests and cultures. Example: When learning about the significance of a z-test, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn to construct a chi-squared test by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Access to tools such as tables, graphs and charts to solve problems
- Develop a reference document with students with verbal and pictorial descriptions.
- Provide students with a graphic organizer that outlines the possible solution paths, formulas and sample problems to facilitate independence.
- Encourage students to verbalize their thinking while solving problems by asking, assessing and advancing questions.

ELL:

- Provide students with visuals aids like pictures and diagrams to illustrate the parts of an expression.
- Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary.
- Identify key phrases or new vocabulary to pre-teach.
- Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to

○ Provide students with reference sheets/notes to encourage confidence and independence.	interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference. ○ Create an outline that allows students to organize and follow information that they are receiving. Outlines can be blank or partially filled in to vary difficulty.

Enrichment

- Extended learning goals:
  - ⇒ Extend WS from Resource sets
  - ⇒ Sociology/Business: Use hypothesis testing to test the claim that more people prefer Pepsi over Coke in a blind taste test. Interpret the results.

Appendix A: Culturally Relevant Pedagogy Examples

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

CONTENT INTEGRATION	KNOWLEDGE CONSTRUCTION	PREJUDICE REDUCTION	EQUITABLE PEDAGOGY	EMPOWERING SCHOOL CULTURE
Teachers use examples and content from a variety of cultures & groups.	Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.	Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.	Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.	Using the other four dimensions to create a safe and healthy educational environment for all.
This unit / lesson is connected to other topics explored with students. There are multiple viewpoints reflected in the content of this unit / lesson. The materials and resources are reflective of the diverse identities and experiences of students. The content affirms students, as well as exposes them to experiences other than their own.	This unit / lesson provides context to the history of privilege and oppression. This unit / lesson addresses power relationships. This unit / lesson help students to develop research and critical thinking skills. This curriculum creates windows and mirrors* for students.	This unit / lesson help students question and unpack biases & stereotypes. This unit / lesson help students examine, research and question information and sources. The curriculum encourage discussion and understanding about the groups of people being represented. This unit / lesson challenges dominant perspectives.	The instruction has been modified to meet the needs of each student. Students feel respected and their cultural identities are valued. Additional supports have been provided for students to become successful and independent learners. Opportunities are provided for student to reflect on their learning and provide feedback.	There are opportunities for students to connect with the community. My classroom is welcoming and supportive for all students? I am aware of and sensitive to the needs of my students and their families. There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Developed by Karla E. Vigil. Adapted with permission from James A. Banks, CULTURAL DIVERSITY AND EDUCATION: FOUNDATIONS, CURRICULUM, AND TEACHING (6th edition), New York: Routledge, 2016, page 5 and Gordon School Institute on Multicultural Practice.

Appendix B: English Language Learners

WIDA Levels:

At the given level of English language proficiency, English language learners will process, understand, produce or use

6- Reaching	• Specialized or technical language reflective of the content areas at grade level • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level • Oral or written communication in English comparable to proficient English peers
5- Bridging	• Specialized or technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports • Oral or written language approaching comparability to that of proficient English peers when presented with grade level material.
4- Expanding	• Specific and some technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs • Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support
3- Developing	• General and some specific language of the content areas • Expanded sentences in oral interaction or written paragraphs • Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support
2- Beginning	• General language related to the content area • Phrases or short sentences • Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support
1- Entering	• Pictorial or graphic representation of the language of the content areas • Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support

Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

Sensory Supports*	Graphic Supports*	Interactive Supports*	Verbal and Textual Supports
• Real-life objects (realia) or concrete objects • Physical models • Manipulatives • Pictures & photographs • Visual representations or models such as diagrams or drawings • Videos & films • Newspapers or magazines • Gestures • Physical movements • Music & songs	• Graphs • Charts • Timelines • Number lines • Graphic organizers • Graphing paper	• In a whole group • In a small group • With a partner such as *Turn-and-Talk* • In pairs as a group (first, two pairs work independently, then they form a group of four) • In triads • Cooperative learning structures such as *Think-Pair-Share* • Interactive websites or software • With a mentor or coach	• Labeling • Students' native language • Modeling • Repetitions • Paraphrasing • Summarizing • Guiding questions • Clarifying questions • Probing questions • Leveled questions such as *What? When? Where? How? Why?* • Questioning prompts & cues • Word Banks • Sentence starters • Sentence frames • Discussion frames • Talk moves, including *Wait Time*

*from *Understanding the WIDA English Language Proficiency Standards. A Resource Guide*. 2007 Edition.. Board of Regents of the University of Wisconsin System, on behalf of the WIDA Consortium—www.wida.us.

Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvsd. 2016

Appendix C: WIDA ELD Standards Integration

ELD-MA.9-12 Explain Interpretive	Interpret mathematical explanations by • Identifying concept or entity • Analyzing data and owning problem-solving approaches • Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles
ELD-MA 9-12 Explain Expressive	Construct mathematical explanations that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions
ELD-MA.9-12 Argue Interpretive	Interpret mathematics arguments by • Comparing conjectures with previously established results and stated assumptions • Distinguishing correct from flawed logic • Evaluating relationships among evidence and mathematical principles to create generalizations
ELD-MA.9-12 Argue Expressive	Construct mathematics arguments that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions

Appendix D: Differentiated Instruction

Strategies to accommodate based on student individual needs::

1. Time/General
   a. Extra time for assigned tasks
   b. Adjust length of assignment
   c. Timeline with due dates for reports and projects
   d. Communication system between home and school
   e. Provide lecture notes/outline

2. Processing
   a. Extra Response time
   b. Have students verbalize steps
   c. Repeat, clarify or reword directions
   d. Mini-breaks between tasks
   e. Provide a warning for transitions
   f. Partnering

3. Comprehension
   a. Precise processes for balanced math instructional model
   b. Short manageable tasks
   c. Brief and concrete directions
   d. Provide immediate feedback
   e. Small group instruction
   f. Emphasize multi-sensory learning

4. Recall
   a. Teacher-made checklist
   b. Use visual graphic organizers
   c. Reference resources to
   d. promote independence
   e. Visual and verbal reminders
   f. Graphic organizers

5. Assistive Technology
   a. Computer/whiteboard
   b. Tape recorder
   c. Video Tape

6. Tests/Quizzes/Grading
   a. Extended time
   b. Study guides
   c. Shortened tests
   d. Read directions aloud

7. Behavior/Attention
   a. Consistent daily structured routine
   b. Simple and clear classroom rules
   c. Frequent feedback

8. Organization
   a. Individual daily planner
   b. Display a written agenda
   c. Note-taking assistance
   d. Color code materials

Appendix E: Enrichment

What is the purpose of enrichment?

The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.

- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is ...	**Enrichment is not...**
• Planned and purposeful • Different, or differentiated, work – not just more work • Responsive to students’ needs and situations • A promotion of high-level thinking skills and making connections within content • The ability to apply different or multiple strategies to the content • The ability to synthesize concepts and make real world and cross curricular connections • Elevated contextual complexity • Sometimes independent activities, sometimes direct instruction • Inquiry based or open-ended assignments and projects • Using supplementary materials in addition to the normal range of resources • Choices for students • Tiered/Multi-level activities with flexible groups (may change daily or weekly)	• Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment) • Worksheets that are more of the same (busywork) • Random assignments, games, or puzzles not connected to the content areas or areas of student interest • Extra homework • A package that is the same for everyone • Thinking skills taught in isolation • Unstructured free time

Appendix F: Resources

Textbook: Larson, R. & Farber, B. Elementary Statistics: Picturing the World (6e). Boston, Pearson, 2015

Appendix G: Climate Change Curriculum Statement

With the adoption of the 2020 New Jersey Student Learning Standards (NJSLS), New Jersey became the first state in the nation to include climate change across content areas. These standards are designed to prepare students to understand how and why climate change happens, the impact it has on our local and global communities and to act in informed and sustainable ways.

Districts are encouraged to utilize the NJSLS to develop interdisciplinary units focused on climate change that include authentic learning experiences, integrate a range of perspectives and are action oriented. While the 2016 NJSLS-English Language Arts (ELA) and Mathematics do not have specific climate change standards, districts may want to consider how they can design interdisciplinary climate change units that incorporate relevant ELA and mathematics standards.

Components of this are tagged throughout the curriculum as appropriate under the “Related Standards” section in each unit.